編號：_____

問題	1: 10分	2: 10分	3: 10分	4: 10分	5: 20分	總分: 100分
得分						
問題	6: 10分	7: 10分	8: 10分	9: 10分		
得分						

請詳列計算和推導過程書寫於題目下方空白處，並將答案書寫於下方指定處。

1. Given $\mathbf{Y} \sim N_3(\boldsymbol{\theta}, \boldsymbol{\Sigma})$, where
$$\boldsymbol{\Sigma} = \begin{pmatrix} 1 & \rho & 0 \\ \rho & 1 & \rho \\ 0 & \rho & 1 \end{pmatrix},$$
for what value of ρ are $Y_1 + Y_2 + Y_3$ and $Y_1 - Y_2 - Y_3$ statistically independent?

2. Consider the regression model

$$E[y_i] = \beta_0 + \beta_1 x_i + \beta_2(3x_i^2 - 2), \qquad i = 1, 2, 3$$

where $x_i = -1$, $x_2 = 0$, and $x_3 = 1$. Find the least squares estimates of β_0, β_1 and β_2. Show that the least squares estimates of β_0 and β_1 are unchanged if $\beta_2 = 0$.

3. Let y_1, y_2, ..., y_n be a random sample from $N(\theta, \sigma^2)$. Find the linear unbiased estimate of θ with minimum variance.

4. Three parcels are weighted at a post office singly in pairs, and all together, giving weights y_{ijk}, i, j, $k = 0$, 1, the suffix 1 denoting the presence of a particular parcel and the suffix 0 denoting its absence. Find the least squares estimates of the weights.

5. Let $y_{ijk} = \mu_{ijk} + \varepsilon_{ijk}$, where

$$\mu_{ij} = \overline{\mu}_{..} + (\overline{\mu}_{i.} - \overline{\mu}_{..}) + (\overline{\mu}_{ij} - \overline{\mu}_{..})$$
$$= \mu + \alpha_i + \beta_{ij},$$

say, $i = 1, 2, \ldots, I$; $j = 1, 2, \ldots, J$; $k = 1, 2, \ldots, K$, and the ε_{ijk} are independent distributed as $N(0, \sigma^2)$.

(a) Find the least squares estimates of μ, α_i, and β_{ij}, and show that they are mutually statistically independent.

(b) Obtain a test statistic for testing the hypotheses $H_1 : \beta_{ij} = 0$ (all i, j) and $H_2 : \alpha_i = 0$ (all i).

6. Derive the expected mean squares for a two-factor analysis of variance with one observation per cell, assuming that both factors are fixed.

7. Construct a 2^{5-2}_{III} design. Determine the effects that may be estimated if a full fold over of this design is performed.

8. Consider the 2^6 design in eight blocks of eight runs each with $ABCD$, ACE, and $ABEF$ as the independent effects chosen to be confounded with blocks. Generate the design. Find the other effects confound with blocks.

9. In an experiment to assess possible side effects of a new drug, 25 rabbits were treated with the drug. Four blood samples were taken on each rabbit. The first blood sample was taken just before treatment started, and additional blood samples were taken at the end of each of the first 3 weeks of treatment. Measurements on the levels of each of 3 substances were made on each blood sample. Consider a T^2 test of the null hypothesis

$$H_0 : \ (\mu_{10}, \mu_{20}, \mu_{30})' = (\mu_{13}, \mu_{23}, \mu_{33})'$$

where $\mu_0 = (\mu_{10}, \mu_{20}, \mu_{30})'$ represents the mean levels of the 3 substances before treatment started and $\mu_3 = (\mu_{13}, \mu_{23}, \mu_{33})'$ represents the mean levels at the end of the third week of treatment. Give a formula for an appropriate T^2 test statistic. Identify any new notation that you introduce in your answer.