編號：____________________

問題	1: 10分	2: 10分	3: 10分	4: 10分	5: 10分	總分: 100分
得分						
問題	6: 10分	7: 10分	8: 10分	9: 10分	10: 10分	
得分						

請詳列計算和推導過程書寫於題目下方空白處，並將答案書寫於下方指定處。

(Please list the calculation and derivation process in the blank space below the question, including the answer.)

1. Let a random variable X have the pdf

$$f(x) = \frac{1}{\sqrt{2\pi}x} e^{-(\log x)^2/2}, \qquad 0 \le x < \infty,$$

Find all moments and the moment generating function of X.

2. If Z is a standard normal random variable, prove the inequality

$$P(|Z| \geq t) \geq \sqrt{\frac{2}{\pi}} \frac{t}{1+t^2} e^{-t^2/2}.$$

3. If (X, Y) has the bivariate normal pdf

$$f(x, y) = \frac{1}{2\pi(1-\rho^2)^{1/2}} \exp\left(\frac{-1}{2(1-\rho^2)}(x^2 - 2\rho xy + y^2)\right),$$

Find the $\mathrm{Corr}(X, Y)$ and $\mathrm{Corr}(X^2, Y^2)$.

4. Suppose that $U_1,\ U_2,\ \ldots,\ U_n$ are iid uniform$(0,1)$ variables, and let $S_n = \sum_{i=1}^n U_i$. Define the random variable N by

$$N = \min\{k : S_k > 1\}.$$

   (a) Show that $P(S_k \le t) = t^k/k!$.

   (b) Find $E[N]$.

5. Show that if $X_1, \ldots, X_n$ are a random sample from a density f that is unknown, then the order statistics are minimal sufficient.

6. Let X_1, …, X_n be iid $N(\theta, 1)$. Show that the best unbiased estimator of θ^2 is $\overline{X}^2 - (1/n)$. Calculate its variance and Cramer-Rao Lower Bound, and show that it is greater than the Cramer-Rao Lower Bound.

7. Suppose $g(t|\theta) = h(t)c(\theta)e^{w(\theta)t}$ is a one-parameter exponential family for the random variable T. Show that this family has an MLR if $w(\theta)$ is an increasing function of θ. Give three examples of such a family.

8. Let $X_1, \ldots, X_n$ be iid $N(\theta, \sigma^2)$, where θ_0 is a specified value of θ and σ^2 is unknown. We are interested in testing

$$H_0 : \theta = \theta_0 \quad \text{versus} \quad H_1 : \theta \neq \theta_0.$$

(a) Show that the test that rejects H_0 when

$$|\overline{X} - \theta_0| > t_{n-1,\alpha/2}\sqrt{S^2/n}$$

is a test of size α.

(b) Show that the test in part (a) can be derived as an LRT.

9. (a) Derive a confidence interval for a binomial p by inverting the LRT of $H_0\colon p = p_0$ versus $H_1\colon p \neq p_0$.

   (b) Show that the interval is a highest density region.

10. Suppose that X_1, …, X_n are iid Poisson(λ). Find the best unbiased estimator of

    (a) $e^{-\lambda}$, the probability that $X = 0$.

    (b) $\lambda e^{-\lambda}$, the probability that $X = 1$.