

編號: _____

問題	1: 10分	2: 10分	3: 10分	4: 10分	5: 10分	總分: 100分
得分						
問題	6: 10分	7: 10分	8: 10分	9: 10分	10: 10分	
得分						

請詳列計算和推導過程書寫於題目下方空白處，並將答案書寫於下方指定處。

1. Let X_1, X_2, X_3 and X_4 be independent continuous random variables with a common distribution function F and let

$$p = P\{X_1 < X_2 > X_3 < X_4\}$$

- (a) Argue that the value of p is the same for all continuous distribution functions F .
- (b) Find p by integrating the joint density function over the appropriate region.

答案: _____

2. Let Y be a random variable and m be a median of Y , i.e., $P(Y \leq m) \geq 1/2$ and $P(Y \geq m) \geq 1/2$. Show that, for any real numbers a and b such that $m \leq a \leq b$ or $m \geq a \geq b$, $E|Y - a| \leq E|Y - b|$.

答案: _____

3. Let $X_i, i = 1, 2, 3$, be independent random variables having the same Lebesgue density $f(x) = e^{-x}I_{(0,\infty)}(x)$. Obtain the joint Lebesgue density of (Y_1, Y_2, Y_3) , where $Y_1 = X_1 + X_2 + X_3, Y_2 = X_1/(X_1 + X_2)$, and $Y_3 = (X_1 + X_2)/(X_1 + X_2 + X_3)$. Are Y_i 's independent?

答案: _____

4. Evaluate

$$\lim_{n \rightarrow \infty} e^{-n} \sum_{k=0}^n \frac{n^k}{k!}$$

答案: _____

5. Let ϕ be a characteristic function and G be a cumulative distribution function on the real line. Show that $\int \phi(ut)dG(u)$ is a characteristic function on the real line.

答案: _____

6. Let ϕ_n be the characteristic function of a probability measure $P_n, n = 1, 2, \dots$. Let $\{a_n\}$ be a sequence of nonnegative numbers with $\sum_{n=1}^{\infty} a_n = 1$. Show that $\sum_{n=1}^{\infty} a_n \phi_n$ is a characteristic function and find its corresponding probability measure.

答案: _____

7. Let F be a cumulative distribution function on the real line \mathbb{R} and $a \in \mathbb{R}$. Show that

$$\int [F(x+a) - F(x)] dx = a.$$

答案: _____

8. Coin 1 comes up heads with probability 0.6 and coin 2 with probability 0.5. A coin is continually flipped until it comes up tails, at which time that coin is put aside and we start flipped the other one.
- (a) What proportion of flips use coin 1?
 - (b) If we start the process with process with coin 1 what is the probability that coin 2 is used on the fifth flip?

答案: _____

9. Customers arrive at a two-server service station according to a Poisson process with rate λ . Whenever a new customer arrives, any customer that is in the system immediately departs. A new arrival enters service first with server 1 and then with server 2. If the service times at the servers are independent exponentials with respective rates μ_1 and μ_2 , what proportion of entering customers completes their service with server 2?

答案: _____

10. Consider a miner trapped in a room which contains three doors. Door 1 leads him to freedom after two days of travel; door 2 returns him to his room after a four-day journey; and door 3 returns him to his room after a six-day journey. Suppose at all times he is equally likely to choose any of the three doors, and let T denote the time it takes the miner to become free. Find $E[T]$.

答案: _____