# Ph.D. Qualifying Examination
# Matrix Theory
# Sep. 13, 2007

*Please write down all the detail of your computation and answers.*

1. (15%) Find the similarity transformation to convert the matrix

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 & 0 \end{pmatrix}$$

   to its Jordan canonical form.

2. (15%) Find permutation matrix $P$, lower triangular matrix $L$ and upper triangular matrix $U$ such that

$$\begin{pmatrix} 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{pmatrix} = PLU.$$

3. (15%) Let $m \times n$ matrix $A$ be the matrix representation of linear transformation $L$. Show that (1) $L$ is one-to-one $\iff$ all columns of $A$ are linearly independent, (2) $L$ is onto $\iff$ all rows of $A$ are linearly independent.

4. (15%) Prove that a complex square matrix is unitarily diagonalizable $\iff$ it is normal.

5. (15%) Show that for $n \times n$ matrix $A$, the induced matrix 2-norm can be computed by

$$\|A\|_2 = \sigma_1(A) = \sqrt{\rho(AA^*)} = \sqrt{\rho(A^*A)},$$

   where $\sigma_1(A)$ and $\rho(A)$ are the largest singular value and the spectral radius of $A$, respectively.

6. (15%) Let $n \times n$ matrix $A$ have all entries 1. Find all of its eigevalues and corresponding eigenvectors.

7. (10%) Find the projection matrix onto the plane $x - y + z = 0$ in $\mathbf{R}^3$.