Qualified Examination: Mathematical Programming
September 2009

1. Solve the following problem:

$$\begin{array}{rl} \text{Maximize} & 4x_1 + 5x_2 - 3x_3 \\ \text{Subject to} & x_1 + x_2 + x_3 = 10 \\ & x_1 - x_2 \geq 1 \\ & x_1 + 3x_2 + x_3 \leq 14 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

2. Solve the following problem by big-M method:

$$\begin{array}{rl} \text{Maximize} & x_1 + 4x_2 - x_4 \\ \text{Subject to} & -x_1 + 2x_2 + -x_3 + x_4 \leq 2 \\ & 2x_1 + x_2 + 2x_3 - 2x_4 = 4 \\ & x_1 - 3x_3 + x_4 \geq 2 \\ & x_1, x_2, x_4 \geq 0 \\ & x_3 \qquad \text{unrestricted} \end{array}$$

3. Let $f : S \subset R^n \to R$ where S is a nonempty convex set. Suppose that x is a local optimal solution to the problem to minimize $f(u)$ subject to $u \in S$. Prove that if f is strictly quasiconvex at x, then x is a global optimal solution. Also prove that if f is strongly quasiconvex at x, then x is the unique global optimal solution.

4. Let X be a nonempty convex set in R^n, and $f : R^n \to R$, $g : R^n \to R^m$ be convex, and $h : R^n \to R^l$ be affine. If $\bar{x}$ is an optimal solution to the problem to minimize $f(x)$ subject to $g(x) \leq 0$, $h(x) = 0$, $x \in X$, then there exist $(\bar{u_0}, \bar{u}, \bar{v}) \neq 0$, $(\bar{u_0}, \bar{u}) \geq 0$ such that:

$$\phi(\bar{u_0}, u, v, \bar{x}) \leq \phi(\bar{u_0}, \bar{u}, \bar{v}, \bar{x}) \leq \phi(\bar{u_0}, \bar{u}, \bar{v}, x)$$

for all $u \geq 0$, $v \in R^l$ and $x \in X$, where $\phi(u_0, u, v, x) = u_0 f(x) + u^t g(x) + v^t h(x)$.