Qualified Examination: Mathematical Programming
February 2009

1. Solve the following problem:

$$\begin{array}{rl} \text{Maximize} & 6x_1 + 4x_2 + 2x_3 \\ \text{Subject to} & 4x_1 - 3x_2 + x_3 \leq 8 \\ & 3x_1 + 2x_2 + 4x_3 \leq 10 \\ & 0 \leq x_1 \leq 3 \\ & 0 \leq x_2 \leq 2 \\ & 0 \leq x_3 \end{array}$$

2. Solve the following problem:

$$\begin{array}{rl} \text{Minimize} & \frac{x_1+3x_2+3}{2x_1+x_2+6} \\ \text{Subject to} & 2x_1 + x_2 \leq 12 \\ & -x_1 + 2x_2 \leq 4 \\ & x_1, x_2 \geq 0 \end{array}$$

3. Let $f : R^n \to R$ be defined by $f(x) = x^t Hx$ where H is an $n \times n$ matrix. The function f is said to be positive subdefinite if $x^t Hx < 0$ implies $Hx \geq 0$ or $Hx \leq 0$ for each $x \in R^n$. Prove that f is quasiconvex on the nonnegative orthant if and only if it is positive subdefinite.

4. Let $f : R^n \to R$ be convex. Show that ξ is a subgradient of f at u if and only if the hyperplane $\{(x, y) : y = f(u) + \xi^t(x - u)\}$ supports epi f at $[u, f(u)]$.