Qualified Examination: Mathematical Programming
September 2007

1. Solve the following problem:

$$\begin{array}{rl} \text{Maximize} & 4x_1 + 5x_2 + 7x_3 - x_4 \\ \text{Subject to} & x_1 + x_2 + 2x_3 - x_4 \geq 1 \\ & 2x_1 - 6x_2 + 3x_3 + x_4 \leq -3 \\ & -2x_1 + 4x_2 + 3x_3 + 2x_4 = -5 \\ & x_1, x_2, x_4 \geq 0 \\ & x_3 \quad \text{unrestricted} \end{array}$$

2. Solve the following problem:

$$\begin{array}{rl} \text{Minimize} & x_1^2 - x_1x_2 + 2x_2^2 - 4x_1 - 5x_2 \\ \text{Subject to} & x_1 + 2x_2 \leq 6 \\ & x_1 \leq 2 \\ & x_1, x_2 \geq 0 \end{array}$$

3. Let $f : R^n \to R$ be defined by $f(x) = x^tHx$ where H is an $n \times n$ matrix. The function f is said to be positive subdefinite if $x^tHx < 0$ implies $Hx \geq 0$ or $Hx \leq 0$ for each $x \in R^n$. Prove that f is quasiconvex on the nonnegative orthant if and only if it is positive subdefinite.

4. Use the Kuhn-Tucker conditions to prove Farkas' theorem.