Qualifying Examination in Discrete Mathematics

for the Ph. D. Program September 2012

Note: The proofs and statements must be detailed. When you quote some theorems, please prove them.

1. Prove that if G is a simple graph with maximum degree 3, then the chromatic index of G is less than or equals to 4. (20%)

2. Find the number of rooted trees with the vertex set $\{1,2,\ldots,9\}$. (20%)

3. Let n and m be positive integers with $n \geq m$. Prove the binomial equation: $\sum_{k=0}^{m} \binom{m}{k}\binom{n}{m-k} = \binom{m+n}{m}$ by two different ways. (20%)

4. Let n be a positive integer, and S be the set of all 2×n matrices $A=[a_{ij}]$ with $\{a_{11},a_{12},\ldots,a_{1n}\}=\{a_{21},a_{22},\ldots,a_{2n}\}=\{1,2,\ldots,n\}$ and $a_{1k}\neq a_{2k}$ for $k=1,2,\ldots,n$. Find the cardinality of S. (20%)

5. True or False.(If true, prove it; if false, give a counterexample and your explanation.)
   (a) If G is a 3-regular graph with no cut-edge, then the chromatic index of G is 3. (10%)
   (b) If G is a 3-connected graph, then G is 3-edge-connected. (10%)