Qualifying Examination in Discrete Mathematics
for the Ph. D. Program September 2011

Note: The proofs and statements must be detailed. When you quote some theorems, please prove them.

1. Let G be a bipartite graph. Give a necessary and sufficient condition for G having a perfect matching and prove it. (15%)

2. Let $V=\{1,2,\ldots,n\}$. Prove that there are n^{n-2} trees with vertex set V. (20%)

3. Let G be a bipartite graph with maximum degree k. Prove that the chromatic index of G is k. (15%)

4. Let G be a simple graph with n vertices $v_1,v_2,\ldots,v_n$ and $\deg(v_i)=d_i$ with $d_1\leq d_2\leq \ldots\leq d_n$. Prove that if $i < n/2$ implies $d_i > i$ or $d_{n-i}\geq n-i$, then G has a Hamiltonian cycle. (20%)

5. Let $f(x)$ be the chromatic polynomial of a graph G of order n. Prove that the number of acyclic orientations of G is $(-1)^n f(-1)$. (15%)

6. Let n be a positive integer. Evaluate the sum $1\cdot2\cdot3+ 2\cdot3\cdot4+\ldots+ (n-1)n(n-1)$. (15%)