

**Qualifying Examination in Discrete Mathematics
for the Ph. D. Program**

September 2010

Note: The proofs and statements must be detailed. When you quote some theorems, please prove them.

1. Suppose G is a planar graph with the girth at least 4. Prove G has a proper 4-coloring. (15%)
2. Suppose n is a positive integer. Find the chromatic index of the complete graph K_n . (15%)
3. Let k be a positive integer. Prove that every k -regular bipartite graph has a perfect matching. (15%)
4. A graph $G=(V,E)$ with n vertices is Hamiltonian-connected if every two distinct vertices of G are connected by a Hamiltonian path. Prove that if $|E|\geq(n-1)(n-2)/2+3$ then G is Hamiltonian-connected. (15%)
5. Prove that if G is 2-connected if and only if every two vertices of G contained in some cycle of G . (15%)
6. Let P_9 be the set of all permutations on $\{1,2,3,\dots,9\}$ and the set $T=\{(x_1,x_2,\dots,x_9)\in P_9 : x_1\neq 1,2, x_2\neq 3, x_3\neq 3,4,5, x_4\neq 5,6, x_5\neq 6,7, x_6\neq 6, \text{ and } x_7\neq 7, x_8\neq 8, x_9\neq 9\}$. Find the cardinality of the set T . (15%)
7. Suppose that a student plans to visit exactly 31 towns in 19 days and visits at least one town in every day. Prove that he will visit exactly 6 towns in some period of consecutive days. (10%)