PhD Qualifying Exam : Discrete Mathematics, September 2007

1 (15 points). Prove that an r-regular bipartite graph can be partitioned into the union of a family of k-regular bipartite subgraphs if and only if k is a factor of r.

2 (15 points). Prove Brooks Theorem: If G is not an odd cycle and not a complete graph, then $\chi(G) \le \Delta(G)$.

3 (14 points) Prove that a cubic graph G has a nowhere zero 3-flow if and only if G is bipartite.

4 (14 points) Prove that if G is k-connected and has no independent set of cardinality k, then G has a hamilton cycle.

5 (14 points) Prove that any bipartite planar graph has a vertex of degree at most 3.

6. (14 points) Prove that any graph G has a subgraph H which is bipartite and $|E(H)| \ge |E(G)|/2$.

7. (14 points) Prove that a graph G is k-colourable if and only if G has an acyclic orientation such that for each cycle C of G, $|C|/|C^+| \le k$ and $|C|/|C^-| \le k$, where C^+, C^- is the set of forward edges and backward edges of C, respectively.