Differential Equations
Ph.D. Qualifying Examination
National Sun Yat-sen University
February, 2018

Answer all the problems below in detail

1.(20 points) Show the solution of Airy's equation $u''(t) + tu(t) = 0$ is oscillatory.

2.(20 points)Assume $u \in C^2(\Omega) \bigcap C^0(\Omega)$ and $\Delta u \geq 0$ in Ω, where Ω is a bounded domain with smooth boundary. Show that either $u = constant$ or $u(\xi) < \max_{\partial\Omega} u$ for all $\xi \in \Omega$.

3.(20 points) Suppose $u = u(x,t)$, and u satisfies

$$\begin{cases} \dfrac{\partial^2 u(x,t)}{\partial t^2} = c^2 \dfrac{\partial^2 u(x,t)}{\partial x^2}, \quad 0 < x < 1, t > 0 \\ u(0,t) = u(1,t) = 0, \quad t > 0 \\ u(x,0) = x(1-x), \quad 0 < x < 1 \\ \dfrac{\partial u(x,0)}{\partial t} = 0, \quad 0 < x < 1 \end{cases} \tag{1}$$

Find the explicit solution for $u(x,t)$.

4.(20 points) Show the system

$$\begin{cases} x' = y \\ y' = -x + y(1 - x^2 - 2y^2) \end{cases} \tag{2}$$

is positively invariant in the annulus $\frac{1}{2} < x^2 + y^2 < 1$ and there exists at least one periodic in the annulus.

5. (20 points) Let $u = u(x,y,t)$ be a twice continuously differentiable solution of

$$\begin{cases} u_t = \Delta u - u^3 \text{ in } \Omega \subset \mathbb{R}^2, \quad t \geq 0, \\ u(x,y,0) = 0 \text{ in } \Omega, \\ u(x,y,t) = 0 \text{ in } \partial\Omega, \quad t \geq 0, \end{cases} \tag{3}$$

where Ω is a bounded domain with smooth boundary. Prove that $u(x,y,t) \equiv 0$ in $\Omega \times [0,T]$.

End of Paper