1. (15 points) Prove that every principal ideal domain is a unique factorization domain.

2. (15 points) Suppose we want to make a necklace consisting of 15 beads, and each bead is of color red or black. How many different kinds of necklace we can make ? (A formula is enough.)

3. (10 points) Suppose $F$ is a field and $f(x), g(x) \in F[x]$ are relatively prime. Prove that there exist polynomials $h(x)$ and $k(x)$ in $F[x]$ such that $f(x)h(x) + g(x)k(x) = 1$.

4. (10 points) Show that $z^4 + 1$ is reducible over $Z_p$ for every prime $p$.

5. (10 points) Suppose $G$ is a finite Abelian group with an odd number of elements. Prove that the product of all elements of $G$ is equal to the identity.

6. (10 points) Let $G$ be a group of order $p^2q^2$, where $p, q$ are primes, $q \not| p^2 - 1$ and $p \not| q^2 - 1$. Prove that $G$ is Abelian.

7. (10 points) Find the minimal polynomial for $\sqrt{-5} + \sqrt{3}$ over $Q$.

8. (10 points) Let $E$ be the splitting field of $x^4 + 1$ over $Q$. Find $G(E/Q)$ and all subfields of $E$.

9. (10 points) Suppose $G$ is an Abelian group of order 16, which contains an element of order 8 and two element of order order 2. Determine the group $G$.