

# 國立中山大學108學年度寒假轉學考招生考試試題

科目名稱：微積分【應數系二年級】

※本科目依簡章規定「不可以」使用計算機

題號：724001

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1. (20%) Find the integrals.

$$A := \int \cos 2x \, dx, \quad B := \int \cos^2 2x \, dx, \quad D := \int \cos^3 2x \, dx.$$

2. (10%) Find the integral  $\int \frac{2x^2 - 1}{3x(x^2 - 1)} \, dx$ .

3. (10%) Find  $\int \frac{1}{\tan x + 1} \, dx$ .

4. (10%) Find all critical points of  $f(x, y) = x^3 - y^3 + 6xy$  and classify each as a relative maximum, relative minimum, or saddle point by using the second-derivatives test.

5. (15%) Find the value of  $\int_0^1 \int_0^1 \sqrt{x+y}(y-2x)^2 \, dx \, dy$ .

6. (10%) Find the range of  $x$  in  $\mathbb{R}$  so that  $\sum_{n=1}^{\infty} \frac{(-10)^n x^n}{4^{2n+1}(n+1)}$  is convergent.

7. (15%) Use Lagrange multiplier to find the minimal value of  $f(x, y) = x + y^2 - xy^2$  under the constraint  $2x^2 + y^2 = 1$ .

8. (10%) An aluminium can is designed to be a round cylinder with height  $h$ , and both the top and bottom are disks of radius  $r$ .

(a) Please find the total area of its surface in terms of  $r$  and  $h$ .

(b) When the total area of its surface is fixed, what is the ratio of  $\frac{h}{r}$  to maximize the volume of this can?