國立中山大學 108 學年度寒假轉學考招生考試試題

科目名稱：線性代數【應數系二年級】　　題號：724002

※本科目依簡章規定「不可以」使用計算機　　共 1 頁第 1 頁

1. [10%] Let $\mathcal{M}_{n\times n}$ be the space of all $n \times n$ real matrices and A a matrix in $\mathcal{M}_{n\times n}$. Define the set $S = \{X \in \mathcal{M}_{n\times n} : AX = O\}$, where O is the $n \times n$ zero matrix. Determine whether S is a subspace in $\mathcal{M}_{n\times n}$ or not, and justify your answer.

2. [15%] Let
$$A = \begin{bmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$
Find the minimum value of the length $|A\mathbf{v} - \mathbf{b}|$, where $\mathbf{v}$ can be any vector in $\mathbb{R}^2$.

3. [15%] Let
$$A = \begin{bmatrix} 1 & 0 & -2 & 1 & -1 \\ 3 & 1 & -2 & 1 & 0 \\ -14 & -5 & 8 & -4 & 0 \\ 73 & 25 & -46 & 23 & -2 \end{bmatrix}$$
and V the space spanned by the columns of A. Find a basis of V.

4. [20%] Let f be a linear map. Show that f is injective if and only if the kernel of f is $\{\mathbf{0}\}$.

5. [20%] Let A be an $n \times n$ matrix. Let A_i be the $(n-1) \times (n-1)$ matrix obtained from A by removing the i-th row and the i-th column. Show that
$$\frac{d}{dx}\det(A + xI_n) = \sum_{i=1}^{n} \det(A_i + xI_{n-1}),$$
where I_k is the $k \times k$ identity matrix.

6. [20%] Suppose $a_0 = 1$, $b_0 = 0$ and a_n, b_n follow the recurrence relation
$$a_{n+1} = a_n + 2b_n$$
$$b_{n+1} = 2a_n + b_n$$
for any $n > 0$. Solve the general forms of a_n and b_n.