國立中山大學 107 學年度寒假轉學考招生考試試題

科目名稱：線性代數【應數系二年級】　　題號：724002

※本科目依簡章規定「不可以」使用計算機　　共 1 頁第 1 頁

1. [10%] Let $S = \{v_1, \ldots, v_n\}$ be a set of vectors in $\mathbb{R}^n$. What is the definition of that S is linearly independent?

2. [15%] Let
$$A = \begin{bmatrix} 1 & -2 & 2 & 3 \\ 4 & -8 & 5 & 6 \\ 7 & -14 & 8 & 10 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & a & 4 & 7 \\ 2 & b & 5 & 8 \\ 3 & c & 6 & 10 \end{bmatrix}.$$
Suppose A and B have the same reduced echelon form. Find a, b, and c.

3. [15%] Let J_5 be the 5×5 all-ones matrix. That is, every entry of J_5 is 1. Find the eigenvalues of J_5, including the multiplicities.

4. [20%] Let $\mathcal{P}_3$ be the vector space of all real-coefficient polynomials of degree at most 3. Then $\mathcal{B} = \{1, x, x^2, x^3\}$ is a basis of $\mathcal{P}_3$. Let T be the differential operator that sends a polynomial $p(x) \in \mathcal{P}_3$ to its derivative $p'(x) \in \mathcal{P}_3$. Show that T is a linear transformation and find the matrix representation of T with respect to the basis $\mathcal{B}$.

5. [20%] Let $\mathcal{S}_3$ be the set of all 3×3 real symmetric matrices. Consider $\mathcal{S}_3$ as a vector space over $\mathbb{R}$. Let $J_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. Show that
$$V = \{X \in \mathcal{S}_3 : J_3 X = X J_3\}$$
is a subspace of $\mathcal{S}_3$. Find a basis of V.

6. [20%] Let A be an $n \times n$ matrix and J_n the $n \times n$ all-ones matrix. Let $A(i,j)$ be the $(n-1) \times (n-1)$ matrix obtained from A by deleting the i-th row and the j-th column. Define
$$\operatorname{cof}(A) = \sum_{\substack{i=1,\ldots,n \\ j=1,\ldots,n}} (-1)^{i+j} \det A(i,j).$$
Show that
$$\det(A + J_n) = \det(A) + \operatorname{cof}(A).$$