Blowup behavior of solutions of nonlinear parabolic equations

Abstract:

We discuss the blowup behavior for solutions of the semilinear equation

$$
\begin{aligned}
&u_t = \Delta u + |u|^{p-1}u \qquad &\text{in} \qquad &\Omega \times (-1, 0)\\
&u(x,t) = 0 \qquad &\text{on} \qquad &\partial\Omega \times (-1, 0),
\end{aligned}
\tag{0.1}
$$

and the weakly coupled system

$$
\begin{gathered}
u_t = \Delta u + v^p, \quad v_t = \Delta v + u^q \qquad \text{in} \qquad \Omega \times (-1, 0),\\
u = 0, \quad v = 0 \qquad \text{on} \qquad \partial\Omega \times (-1, 0).
\end{gathered}
\tag{0.2}
$$

Suppose that u is a solution of (0.1) and satisfies the blowup rate

$$|u(x,t)| \leq C|t|^{1/(p-1)} \quad \text{in} \quad \Omega \times (-1, 0).$$

When p is subcritical, i.e., $1 < p < (n+2)/n-2)$ if $n \geq 3$ and $1 < p < \infty$ if $n = 1, 2$, we prove that $a \in \Omega$ is a blowup point if and only if

$$\lim_{t \to 0} |t|^{1/(p-1)} \left| u\left(a + |t|^{1/2} y, t\right) \right| = (p-1)^{-1/(p-1)}$$

uniformly for $|y| < C$. Moreover, there is no blowup on the boundary of Ω. Here, the domain Ω is possibly unbounded and not necessarily convex. We also allow u to be vector-valued.

Suppose that (u, v) is a non-negative solution of (0.2). Let $1 < p_0 < \frac{n+3}{n+1}$. When $p \geq 1$, $q \geq 1$, $pq > 1$ and $|p - p_0| + |q - p_0|$ is small, we will prove that $a \in \Omega$ is a blowup point if and only if for each $C > 0$ we have

$$\lim_{t \to 0} |t|^{\frac{p+1}{pq-1}} u\left(a + |t|^{1/2} y, t\right) = \Gamma \quad \text{and} \quad \lim_{t \to 0} |t|^{\frac{q+1}{pq-1}} v\left(a + |t|^{1/2} y, t\right) = \gamma$$

uniformly for $|y| < C$, where

$$\gamma^p = \Gamma \ \frac{p+1}{pq-1}, \qquad \Gamma^q = \gamma \ \frac{q+1}{pq-1}.$$

Moreover, there is no blowup on the boundary of Ω.