Pseudospectral and Runge-Kutta-Nyström methods for second-order wave equations: Stable and accurate boundary treatments

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Many wave phenomena in general relativity , acoustics, and electromagnetic, are described by second-order wave equations. For these wave problems, the domain may be large compared the wavelength, and the waves have to propagate long distances. Hence, numerically solving these problems generally requires long time integration. As a consequence, accumulation of numerical dispersion error may affect the simulation quality. It has be shown that high-order methods are more efficient than the low-order methods in preserving low accumulation of dispersion error during long time integration. However, high-order schemes are very sensitive to the imposition of boundary conditions, and great care must be exercised to ensure stable and accurate computations of high-order schemes.

In this talk we present a high-order scheme based on the pseudospectral Legendre approximation in space and the Runge-Kutta-Nyström algorithm in time, which can be adopted in a multi-domain computational framework, to solve second order wave equations. The key toward to the success of constructing such a stable scheme hinges upon properly imposing penalty boundary conditions at every collocation equations. We shall use one dimensional space problems to illustrate the conceptual ideas of the methods, and special attention is paid to analyzing the stability of the scheme subject to various types of boundary conditions, including Dirichlet, Neumann, Robin, and material interface boundary conditions. Through conducting energy estimates it is shown that the scheme can be made stable by properly choosing the penalty parameters. The present one-dimensional scheme can be generalized for multidimensional problems even defined on curvilinear coordinates. Numerical experiments for model problems defined on one and two dimensional spaces are conducted, and we observe the expected convergence results.