國立中山大學 105 學年度寒假轉學考招生考試試題

科目名稱：線性代數【應數系二年級】　　　　題號：20402

※本科目依簡章規定「不可以」使用計算機　　　　共 1 頁第 1 頁

1. [10%] Compute the determinant of the matrix

$$\begin{pmatrix} -2 & 5 & -1 & 3 \\ 1 & -9 & 13 & 7 \\ 3 & -1 & 5 & -5 \\ 2 & 8 & -7 & -10 \end{pmatrix}$$

and determine whether it is invertible.

2. [15%]Prove that there is a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ satisfying $T(1,3) = (1,2,3)$ and $T(-2,1) = (4,5,6)$. Is T unique? What is $T(3,4)$? Verify your answers.

3. [15%] Find the Jordan canonical form of the matrix

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 2 & 0 \\ -2 & -2 & -1 \end{pmatrix}.$$

4. [15%] Find all values of k so that the set $\{(1,-3,1),(3,4,5),(2,1,k)\}$ of vectors in $\mathbb{R}^3$ is linearly dependent.

5. [15%] Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation satisfying $T^2 = I$, the identity transformation. Find the range and kernel of T.

6. [30%] For any $n \in \mathbb{N} \cup \{0\}$, let $P_n(\mathbb{R})$ be the set of polynomials over $\mathbb{R}$ with degrees less or equal to n. Define a transformation $T : P_2(\mathbb{R}) \to P_3(\mathbb{R})$ by

$$T(p(x)) = xp'(x) + \int_0^x p(t)\,dt,$$

where $'$ is the first order derivative.

   (1) Show that $P_3(\mathbb{R})$ is a vector space.

   (2) Prove that T is linear.

   (3) Find the matrix representing T with respect to the ordered basis $\{1, x, x^2\}$ and $\{1, x, x^2, x^3\}$ for $P_2(\mathbb{R})$ and $P_3(\mathbb{R})$, respectively.