

國立中山大學 106 學年度寒假轉學考招生考試試題

科目名稱：微積分【應數系二年級】

題號：20401

※本科目依簡章規定「不可以」使用計算機

共 1 頁第 1 頁

請詳列計算證明之過程。

1. (a) State the definition of $\lim_{x \rightarrow a} f(x) = L$ where $a, L \in \mathbf{R}$. (10%)
(b) Show that $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$ provided both limits exist. (10%)
2. (a) Find equations of the tangent lines to the graph of $f(x) = -x^2 + 4x + 1$ that pass through $(2, 6)$. (10%)
(b) Find all relative extreme values of $f(x) = 2x^3 + 3x^2 - 12x + 1$. (5%)
3. (a) Evaluate $\int_0^2 \frac{1}{e^x + e^{-x}} dx$. (10%)
(b) Find $\int \frac{2}{1-x^2} dx$. (5%)
4. (a) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} nx^n$. (10%)
(b) Find the sum of $\sum_{n=1}^{\infty} nx^n$ for those x inside the interval of convergence. (10%)
5. (a) Find all second partial derivatives of $f(x, y) = xe^{\frac{x}{y}}$. (5%)
(b) Find all relative extreme values of $f(x, y) = x^2 - 6x + 2y^2 + 8y + 2$. (5%)
6. (a) Let $E = \{(x, y) \mid -2 \leq x \leq 2, 0 \leq y \leq 2, 1 \leq x^2 + y^2 \leq 4\}$. Evaluate the double integral $\iint_E x^2 y dA$. (10%)
(b) Evaluate the triple integral $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} y dz dy dx$. (10%)