國立中山大學 106 學年度寒假轉學考招生考試試題

科目名稱：高等微積分【應數系三年級】　　題號：20403
※本科目依簡章規定「不可以」使用計算機　　共 1 頁第 1 頁

1. [15%] Let $\{a_n\}$ be a sequence of complex numbers satisfying

$$|a_{n+1} - a_n| \le 3^{-n}$$

for all n. Determine whether the sequence $\{a_n\}$ has a limit, and verify your assertion.

2. [15%] State the definition of the closure of a subset of $\mathbb{R}$, and use the definition that you provide to find the closure of the set

$$\{n^{-1} : n \in \mathbb{N}\}.$$

Verify your assertion.

3. [15%] Use the ϵ-δ argument to prove that the function $x \mapsto 1/x^2$ is continuous on $(0, \infty)$. Is the function also uniformly continuous on $(0, \infty)$? Verify your assertion.

4. [15%] Show that any function $f : \mathbb{R} \to \mathbb{R}$ satisfying the inequality

$$|f(x) - f(y)| \le |x - y|^2$$

for any real x and y is a constant function.

5. [15%] Find the Taylor series expansion for the function

$$x \mapsto \frac{1}{1+x}$$

at 2, and find the convergence of interval of the Taylor series.

6. [15%] Let $f_n : [1, 2] \to \mathbb{R}$ be defined by

$$f_n(x) = \frac{x}{(1+x)^n}.$$

Show that

$$\sum_{n=1}^{\infty} f_n(x)$$

converges in $[1, 2]$. Is the convergence uniform? Verify your assertion.

7. [10%] Find the limit $n^{1/n}$ as $n \to \infty$.